HW 5 P133

① $$\int_C f\,dz = \int_0^{\pi/2} e^{(-1-2i)\operatorname{Log} e^{i\theta}}\, i e^{i\theta}\, d\theta$$

$$= \int_0^{\pi/2} e^{(2\theta - i\theta)}\, i e^{i\theta}\, d\theta$$

$$= i\left(\frac{e^{\pi}-1}{2}\right)$$

⑧ $$\int_C f\,dz = \int_{-\pi}^{\pi} e^{(a-1)(\log R + i\theta)} R i e^{i\theta}\, d\theta$$

$$= Ri \int_{-\pi}^{\pi} e^{(a-1)\log R + ia\theta}\, d\theta$$

$$= \frac{R^a}{a}\left(e^{ia\pi} - e^{-ia\pi}\right)$$

$$= \frac{2R^a}{a} i \sin a\pi .$$

9a $$\int_C f\,dz = \int_{-\pi}^{\pi} e^{-3/4 \operatorname{Log} e^{i\theta}}\, i e^{i\theta}\, d\theta$$

$$= \int_{-\pi}^{\pi} i e^{i\theta/4}\, d\theta$$

$$= 4\sqrt{2}i .$$

9b $$\int_C f\,dz = \int_0^{2\pi} i e^{i\theta/4}\, d\theta$$

$$= 4(i-1).$$

P138

(1a) $\left|\frac{z+4}{z^3-1}\right| \le \frac{|z|+4}{|z|^3-1} = \frac{6}{7}$ on C

$$\left|\int_C \frac{z+4}{z^3-1}\,dz\right| \le \frac{6}{7}\pi$$

(1b) $\left|\frac{1}{z^2-1}\right| \le \frac{1}{|z|^2-1} = \frac{1}{3}$ on C.

$$\left|\int_C \frac{dz}{z^2-1}\right| \le \frac{\pi}{3}.$$

(2a) (2) According to the suggestion,

$$\left|\int \frac{dz}{z^4}\right| \le \left(\frac{2}{\sqrt{2}}\right)^4 \cdot \sqrt{2} = 4\sqrt{2}$$

(3) $$\left|\int_{C_R} \frac{\operatorname{Log} z}{z^2}\,dz\right| \le \int_{C_R} \frac{|\log R + i\theta|}{R^2}\,dz$$

$$\le \frac{2\pi R}{R^2}\cdot(\log R + \pi)$$

$$= 2\pi\left(\frac{\pi + \log R}{R}\right)$$

$$\lim_{R\to\infty} \frac{\pi + \log R}{R} = \lim_{R\to\infty} \frac{\frac{1}{R}}{1} = 0.$$

(6) let $M>0$ s.t. $|f| \le M$ in $\{|z| \le 1\}$ since it is analytic, then

$$\left| \int_{C_\rho} z^{-1/2} f(z)\, dz \right| \le \int_{C_\rho} \left| e^{-\frac{1}{2}\log(\rho e^{i\theta})} f(z) \right| dz$$

$$\le 2\pi\rho \cdot M \left| \rho^{-1/2} e^{-\frac{1}{2}i\theta} \right|$$

$$= 2\pi\sqrt{\rho} M \to 0 \text{ as } \rho \to 0.$$

since the bound M is independent of ρ.