

HW 5 P 133

$$\begin{aligned}
 \textcircled{7} \int_c f dz &= \int_0^{\pi/2} e^{(-1-zi) \text{Log} e^{i\theta}} i e^{i\theta} d\theta \\
 &= \int_0^{\pi/2} e^{(2\theta - i\theta)} i e^{i\theta} d\theta \\
 &= i \left(\frac{e^\pi - 1}{2} \right)
 \end{aligned}$$

$$\begin{aligned}
 \textcircled{8} \int_c f dz &= \int_{-\pi}^{\pi} e^{(a-1)(\log R + i\theta)} R i e^{i\theta} d\theta \\
 &= R i \int_{-\pi}^{\pi} e^{(a-1) \log R + i a \theta} d\theta \\
 &= \frac{R^a}{a} \left(e^{i a \pi} - e^{-i a \pi} \right) \\
 &= \frac{2 R^a}{a} i \sin a \pi.
 \end{aligned}$$

$$\begin{aligned}
 \textcircled{9a} \int_c f dz &= \int_{-\pi}^{\pi} e^{-3/4 \log e^{i\theta}} i e^{i\theta} d\theta \\
 &= \int_{-\pi}^{\pi} i e^{i\theta/4} d\theta \\
 &= 4\sqrt{2}i.
 \end{aligned}$$

$$\begin{aligned}
 \textcircled{9b} \int_c f dz &= \int_0^{2\pi} i e^{i\theta/4} d\theta \\
 &= 4(i-1).
 \end{aligned}$$

$$(1a) \quad \left| \frac{z+4}{z^3-1} \right| \leq \frac{|z|+4}{|z|^3-1} = \frac{6}{7} \quad \text{on } C$$

$$\left| \int_C \frac{z+4}{z^3-1} dz \right| \leq \frac{6}{7} \pi$$

$$(1b) \quad \left| \frac{1}{z^2-1} \right| \leq \frac{1}{|z|^2-1} = \frac{1}{3} \quad \text{on } C.$$

$$\left| \int_C \frac{dz}{z^2-1} \right| \leq \frac{\pi}{3}.$$

(2) According to the suggestion,

$$\left| \int \frac{dz}{z^4} \right| \leq \left(\frac{r}{\sqrt{2}} \right)^4 \cdot \sqrt{2} = 4\sqrt{2}$$

$$(3) \quad \left| \int_{C_R} \frac{\text{Log } z}{z^2} dz \right| \leq \int_{C_R} \frac{|\log R + i\theta|}{R^2} dz$$

$$\leq \frac{2\pi R}{R^2} (\log R + \pi)$$

$$= 2\pi \left(\frac{\pi + \log R}{R} \right)$$

$$\lim_{R \rightarrow \infty} \frac{\pi + \log R}{R} = \lim_{R \rightarrow \infty} \frac{\frac{1}{R}}{1} = 0.$$

6 let $M > 0$ s.t. $|f| \leq M$ in $\{|z| \leq 1\}$ since it is

analytic, then

$$\left| \int_{C_\rho} z^{-1/2} f(z) dz \right| \leq \int_{C_\rho} \left| e^{-1/2 \log(\rho e^{i\theta})} f(z) \right| dz$$

$$\leq 2\pi\rho \cdot M \left| \rho^{-1/2} e^{-1/2 i\theta} \right|$$

$$= 2\pi\rho M \rightarrow 0 \text{ as } \rho \rightarrow 0.$$

Since the bound M is independent of ρ .

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